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High frequency properties of electron waveguides, AC-admittance, scattering parameters, plasma oscillations

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Abstract. The relation between the AC-admittance of single-mode electron waveguides is discussed in terms of the more complete scattering matrix of the same structure. For low frequencies, the admittance coincides with the series-inductance expressions in the literature. For higher frequencies the admittance shows resonances where it vanishes due to standing plasma waves.

Introduction

Electron waveguide devices are a class of devices thought of as candidates for future high speed electronics. Therefore it is natural that the AC behavior of these structures has attracted increasing attention. In some low frequency calculations [1, 2] reflected and transmitted modes yield capacitive and inductive contributions respectively. For a long single-mode electron waveguide as in Fig. 1(a), where the inductive contribution is dominant, these calculations will typically yield an admittance built up of the usual DC-resistance $R_0 = h/2e^2 \approx 13$ kΩ in series with the kinetic inductance of the electrons in the waveguide yielding

$$Y(\omega) = 1/[R_0 + j\omega l R_0/(2v_\mathrm{F})], \qquad (1)$$

where l is the length of the electron waveguide and v_F is the Fermi velocity.

Another manifestation of the dynamics in a single-mode electron waveguide or quantum wire are the plasma oscillations. These have been studied in for example [3, 4]. In contrast with the admittance calculations above most of these papers only consider infinitely long quantum wires, but on the other hand take the electrostatic environment of the structure into account (dielectric constant of the surrounding media, screening effects, etc).

In [5], quite a different microwave-engineering approach was applied to the plasma waves, thus describing the plasma waves as voltage and current waves traveling on transmission lines. This approach also allowed us to calculate the connection rules at the interfaces to the reservoirs forming part of ordinary microstrip transmission lines. These connection rules were formulated in terms of scattering-matrices.

The model developed in [5] also allows us to calculate the admittance. The object of the present paper is to do this and thus build a bridge between this microwave-engineering model and the concept of AC-admittance and thereby also relate the quantum inductance to the plasma waves. Apart from giving physical insight, this can also be used as a powerful tool for analyzing the hf-properties of electron waveguide devices. As an example we will use a long ideal single-mode electron waveguide at zero
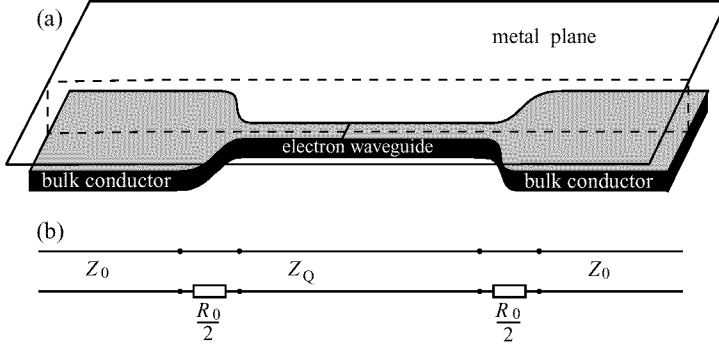
Fig 1. (a) Electron waveguide of length L between two reservoirs, (b) Transmission-line equivalent of the structure.

temperature. For low frequencies, the admittance obtained in this way is the same as the series quantum inductance formula. For higher frequencies, the expressions differ, yielding resonances where the admittance vanishes, since standing wave patterns form in the electron waveguide.

1   The voltage wave scattering matrix of an electron waveguide

We will now consider the electron waveguide as a micro-wave engineering 2-port where incoming voltage waves are partly reflected, partly transmitted to the other port. We will also derive the scattering matrix describing this. In the electron waveguide the signals propagate as plasma waves described in [5]. The propagation velocity of these waves was there found to be $v_\mathrm{p} = \sqrt{1/(L_\mathrm{tot}C_\mathrm{tot})}$ where $L_\mathrm{tot} = L_\mathrm{g} + R_0/(2v_\mathrm{F})$ is the total inductance per unit length which is a sum of the ordinary magnetic inductance L_g and the kinetic inductance $R_0/(2v_\mathrm{F})$ and v_F is the Fermi velocity. In the same manner, the total capacitance per unit length is formed by the ordinary, electrostatic capacitance to the nearby conducting plane C_g and the quantum capacitance due to the reduced density of states $C_\mathrm{tot}^{-1} = C_\mathrm{g}^{-1} + [2/(R_0v_\mathrm{F})]^{-1}$. The boundary condition for the plasma wave at the interface between the electron waveguide and the reservoir (that forms the first part of the connecting microwave transmission line) was also derived. The result was a transmission-line equivalent where a characteristic impedance for the electron waveguide $Z_Q = \sqrt{(L_\mathrm{tot}/C_\mathrm{tot})}$ was introduced and a contact resistance of $R_0/2$ represented the junction. The total transmission-line equivalent for the electron waveguide connected between two microwave transmission lines then consists of a transmission line with Z_Q representing the electron waveguide between the $R_0/2$ resistors connecting to the ordinary transmission lines. See Fig. 1(b). This model then gives the scattering matrix **S** relating ingoing and outgoing waves $\bar{V}^+$ and $\bar{V}^-$ at the structure:

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}. \tag{2}$$

Using standard microwave-engineering analysis **S** is calculated from this model:

$$S_{11} = S_{22} = \frac{\left(\frac{R_0}{2} + Z_Q\right)^2 - Z_0^2 - \left[\left(\frac{R_0}{2} - Z_Q\right)^2 - Z_0^2\right]e^{-2jkl}}{A} \tag{3}$$

$$S_{12} = S_{21} = \frac{4e^{-jkl}Z_0Z_Q}{A} \tag{4}$$

where

$$A = \left[\left(1 + e^{-jkl}\right)\left(\frac{R_0}{2} + Z_0\right) + \left(1 - e^{-jkl}\right)Z_Q \right]$$
$$\times \left[\left(1 - e^{-jkl}\right)\left(\frac{R_0}{2} + Z_0\right) + \left(1 + e^{-jkl}\right)Z_Q \right]. \tag{5}$$

If we choose reasonable values [5] $Z_Q = 2R_0$, $Z_0 = 100\,\Omega$, the reflection S_{11} stays within 1.5% from unity and that the transmission $|S_{12}|$ oscillates between 0.7% and 1.5%. The small transmission is due to the impedance mismatch between the ordinary transmission line and the electron waveguide. The corresponding measurement can be done with a microwave network analyzer. Measuring for example on a split gate structure, the ground of the hf-probes would be put in contact with the gate metal, while the signal leads would be put in contact with the ohmic contacts at the electron reservoirs.

2   Relation between the scattering matrix and the AC-admittance

The S-parameters relate incoming and outgoing voltage waves on the connecting transmission lines and since these waves are related to voltages and currents, we can use the S-parameters to calculate the admittance. Let waves with opposite signs impinge on the two ports $V_1^+ = -V_2^+$. Making use of the symmetry of the device we can calculate the voltage across the structure: $V = V_1 - V_2 = 2(V_1^+ + V_1^-) = 2V_1^+(1 + S_{11} - S_{12})$. In the same manner the current can be written as $I = I_1 = -I_2 = (V_1^+ - V_1^-)/Z_0 = V_1^+(1 - S_{11} + S_{12})/Z_0$. This yields the admittance

$$Y = \frac{I}{V} = \frac{1}{2Z_0}\frac{1 - S_{11} + S_{12}}{1 + S_{11} - S_{12}}. \tag{6}$$

This expression is the admittance in the sense that it relates voltage across the structure with the current into it. Also a measurement set-up with micro strips transmitting the micro-wave signal to the electron-waveguide contacts would yield such an odd excitation ($V_1^+ = -V_2^+$). There is however room for some discussion on the validity of this expression. Note that the net in Fig. 1(b) actually has 4 leads if we count the ground, and since there is a capacitive coupling to the ground it is not self-evident that the above expression is the correct one. Other excitations (e.g. $V_1^+ = V_2^+$) yield other result, but $V_1^+ = -V_2^+$ has the nice property of yielding the same current in port 1 as in port 2.

Inserting the expressions for the scattering parameters (3, 4) into (6) we obtain the admittance

$$Y(\omega) = \frac{1}{[R_0 + j2Z_Q\tan(kl/2)]}. \tag{7}$$

Note that the characteristic impedance Z_0 of the transmission lines used for the measurement cancels. Assuming a linear dispersion for the plasma waves $k = \omega/v_p$ the expression coincides with (1) for low frequencies since $(Z_Q l)/v_p = L_{tot}l \approx lR_0/(2v_F)$ since the magnetic inductance is negligible. Both expressions are illustrated in Fig. 2. We see that the expression derived in this paper oscillates due to the standing plasma
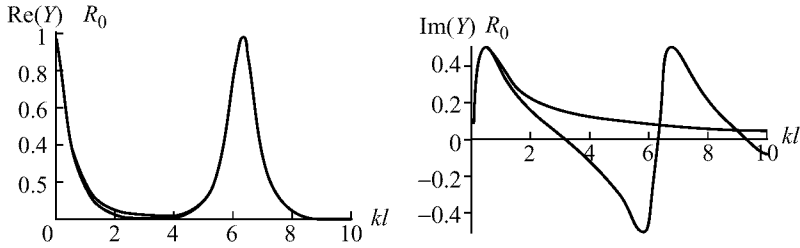
Fig 2. Real and imaginary part of the admittance of the structure in Fig. 1.

waves that form in the electron waveguide. At some frequencies both the real and the imaginary part of the admittance vanish. When kl reaches 2π the curve repeats itself, again yielding a real admittance of $1/R_0$. For a $10\,\mu$ long electron waveguide with a plasma velocity v_{p} as low as 10^5 m/s the deviation from (1) would be seen already at a few GHz.

In conclusion, the relation between the admittance and the plasma oscillations, was investigated using a microwave-engineering approach where the voltage wave scattering matrix was used as an intermediate step. This approach proved to be a useful tool in finding the high frequency admittance of an electron-waveguide structure.

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